# The variance of the length of stay and the optimal DRG outlier payments

Stefan Felder[*,◊]

Otto-von-Guericke University Magdeburg

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**Abstract:** Prospective payment schemes in health care often include supply-side insurance for cost outliers. In the early US Medicare and the current German DRG systems, the outlier scheme fixes a length of stay (LOS) threshold, constraining the loss risk for the provider. This threshold is to increase with the standard deviation of the LOS distribution. The present paper addresses the adequacy of the DRG threshold rule for risk-averse hospitals with preferences depending on expected profits and its variance. It first shows that the optimal threshold solves the hospital's tradeoff between higher profit risk and lower premium loading payments. It then demonstrates for normally distributed LOS that the optimal threshold generally decreases with an increase of the standard deviation. The intuition for this result is that a higher variance increases the profit risk, which in turn leads hospitals to insure a larger part of the LOS distribution.

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**Keywords:** Optimal outlier DRG payments, supply-side insurance in health care, stop loss insurance

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[*] Institute of Social Medicine and Health Economics (ISMHE), Faculty of Medicine, Leipziger Str. 44, 39120 Magdeburg, Germany. Fon +49 391 67 24 321, Fax, +49 391 67 24 310, Email stefan.felder@ismhe.de.

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## 1. Introduction

In the mid-eighties US Medicare introduced the prospective payment system, under which hospital reimbursements for patient discharges were based on their classification into diagnosis related groups (DRGs). Prospective payments replaced the old cost-based reimbursement system which usually depended on a hospital's characteristics and its patients' length of stay (LOS). The change from a retro- to a prospective payment system transferred the loss risk from the insurers to the providers and gave the latter an incentive to economize treatment costs.

Still, Medicare saved part of the former system by introducing outlier payments. For patients staying longer than a stated threshold, hospitals could charge the costs of treatment based on the actual LOS, while for patients discharged within the threshold, pure prospective payment applied. The new scheme resembled an insurance contract with a deductible, i.e. the hospital insures only the part of the LOS distribution beyond the threshold.

Like many other industrialized countries, Germany followed US Medicare and introduced its own DRG payment system for hospitals about twenty years later. Thereby it decided to use the original Medicare outlier method which couples the threshold to the standard deviation of LOS according to[1]:

$$m = \exp\left(\tilde{\mu} + 2\tilde{\sigma}\right) , \qquad (1)$$

where $\tilde{\mu}$ and $\tilde{\sigma}$ is the mean and standard deviation of the log of LOS, respectively.

Table 1 presents characteristics for the top 30 German DRGs in 2005, calculated from a sample of roughly 270,000 hospital cases in the state of Saxony-Anhalt. The average LOS is 7.29 days and its average standard deviation 4.1 days. The next column shows the actual threshold for the individual DRGs as published in 2005. On average the actual threshold is 15.86 days, which is close to two standard deviations above the mean. When I calculate the threshold based on the logs of LOS as required by the DRG outlier methodology, the values in the fifth column arise, the average being 20 days. The last two columns in Table 1 present the effect of an increase of the standard deviation on the hospital's marginal risk (details follow in Section 3), based on the actual and the calculated thresholds, respectively. It shows that for almost all

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[1] The specific rule is $m = \min\left(\exp\left(\tilde{\mu} + 2\tilde{\sigma}\right), \exp\left(\tilde{\mu}\right) + b\right)$ where $b$ is a policy parameter to be set such that the LOS threshold covers between 94 and 95 percent of all cases. For log-normally distributed LOS, the parameter $b$ will determine the threshold, as $\tilde{\mu} + 2\tilde{\sigma}$ covers close to 98 percent of total cases. But since the cumulative density function of the normal distribution is decreasing in $\tilde{\sigma}$ for $m > \mu$ (see section 3), the threshold also increases with $\tilde{\sigma}$ if $b$ is binding.

DRGs, the amount of marginal risk born by the hospital increases when the standard deviation of LOS rises.

**Table 1**: **The top 30 German DRGs (2005)**

| DRG | LOS | | LOS threshold | | The change of risk | |
|---|---|---|---|---|---|---|
| | Mean ($\mu$) | Std. dev. ($\sigma$) | Actual ( $\tilde{m}$ ) | calculated according to (1) ($m$) | $\partial k(\tilde{m})/\partial\sigma$ | $\partial k(m)/\partial\sigma$ |
| B70B | 11.83 | 6.05 | 23 | 29 | 0.20 | 0.05 |
| F62B | 12.47 | 7.14 | 26 | 39 | 0.27 | 0.01 |
| C08Z | 3.12 | 1.88 | 5 | 6 | 0.79 | 0.37 |
| G67C | 4.19 | 3.19 | 8 | 13 | 0.88 | 0.12 |
| F67B | 6.13 | 3.48 | 12 | 19 | 0.33 | 0.01 |
| I68B | 9.23 | 5.10 | 23 | 26 | 0.08 | 0.03 |
| O60C | 3.93 | 2.01 | 7 | 10 | 0.31 | 0.05 |
| F62C | 10.02 | 5.51 | 22 | 30 | 0.17 | 0.01 |
| E71B | 5.96 | 5.04 | 16 | 22 | 0.47 | 0.06 |
| G60B | 2.91 | 2.98 | 10 | 9 | 0.34 | 0.63 |
| B69B | 7.39 | 4.02 | 15 | 21 | 0.23 | 0.02 |
| E77C | 8.98 | 4.98 | 17 | 27 | 0.34 | 0.01 |
| G49Z | 1.61 | 0.70 | n.d | 3 | n.d | 0.07 |
| E77B | 12.05 | 7.29 | 24 | 40 | 0.41 | 0.01 |
| I44Z | 14.29 | 3.99 | 25 | 22 | -0.04 | -0.21 |
| G54Z | 6.90 | 3.80 | 14 | 16 | 0.25 | 0.12 |
| B80Z | 2.96 | 2.59 | 6 | 8 | 1.02 | 0.51 |
| F66B | 5.83 | 4.00 | 13 | 20 | 0.43 | 0.02 |
| D30Z | 6.06 | 2.75 | 11 | 13 | 0.13 | 0.05 |
| E65B | 8.91 | 5.05 | 19 | 27 | 0.23 | 0.01 |
| F49C | 1.83 | 0.53 | n.d | 3 | n.d | -0.07 |
| D63Z | 4.83 | 3.16 | 9 | 14 | 0.66 | 0.09 |
| G24Z | 6.89 | 4.93 | 12 | 17 | 0.92 | 0.35 |
| I48Z | 14.60 | 4.32 | 25 | 23 | -0.07 | -0.18 |
| I69Z | 9.65 | 5.72 | 24 | 35 | 0.13 | 0.00 |
| B76D | 5.11 | 5.62 | 14 | 21 | 0.95 | 0.13 |
| L20Z | 7.02 | 4.71 | 13 | 18 | 0.71 | 0.24 |
| F62D | 8.02 | 4.69 | 19 | 25 | 0.16 | 0.01 |
| G48Z | 9.81 | 5.96 | 20 | 25 | 0.38 | 0.11 |
| G67B | 6.25 | 4.48 | 12 | 19 | 0.77 | 0.11 |
| **Mean** | 7.29 | 4.19 | 15.86 | 19.97 | 0.41 | 0.09 |
| **Max** | 14.60 | 7.29 | 26 | 40 | 1.02 | 0.63 |
| **Min** | 1.61 | 0.53 | 5 | 3 | -0.07 | -0.21 |

Source: own calculation based on 268.977 DRG cases of AOK patients in Saxony-Anhalt, 2005.

A higher risk would indicate that hospitals want to extend the insurance coverage, i.e. to decrease the LOS threshold. In fact this is the main result of this paper, contradicting the actual German DRG outlier methodology.

The health economics literature dealt with insurance aspects of DRG outlier payments early on when Medicare introduced its prospective reimbursement system. *Ellis and McGuire* (1988) applied Arrow's principle that full insurance after a deductible is the optimal structure of insurance (see *Arrow*, 1963) to hospital reimbursement. They show that outlier payments should be based on the hospital's average loss per case rather than on individual case-level losses, since the hospital can itself pool the loss risk of individual cases.

*Keeler et al.* (1988), combining optimal the deductible with coinsurance on the marginal costs of expensive cases to reduce moral hazard, came to the same conclusion. Outlier payments serve as an insurance for hospitals against excessive losses and they mitigate problems of access and underprovision of care for the patients in need of costly treatment. *Keeler et al.* also studied the optimal policy for paying more than one DRG when the outlier payments have to be made case by case. If the hospital's utility is quadratic, they show that the optimal scheme is deductibles that are the same for all DRGs if there are no coinsurance restrictions and a stop equal average loss policy to each DRG per outlier under a constant coinsurance rate. Since with concave utility the marginal value of money is higher when losses are greater, equalizing the expected loss on each DRG by adjusting the deductible correspondingly is the optimal outlier payment policy.

This paper's focus is on the relationship between the LOS standard deviation and the optimal threshold. Section 2 presents the optimal risk sharing between the hospital and the insurer. The existence of a positive threshold arises since I assume loading on the insurance net premium and risk aversion on the part of the hospitals. I derive the optimal threshold for a model that represents the hospital's utility function using the $(\mu, \sigma)$ criterion and comparative-static results regarding the degree of risk aversion, the loading factor and the costs of stay per diem. Section 3 deals with the comparative statics referring to the LOS standard deviation, assuming normally distributed LOS truncated from below at zero. It shows that the optimal threshold usually decreases with an increase of the standard deviation. Section 4 discusses and concludes.

## 2. Optimal risk sharing between the hospital and the insurer

To begin with, I set the costs per diem of a hospital stay equal to one. The insurer is assumed to pay the hospital depending on a patient's LOS *t* according to the following rule:

$$\begin{array}{ll} E^m, & \text{if } t < m \\ t, & \text{if } t \geq m, \end{array} \qquad (2)$$

where $m$ is the outlier threshold ($m > 0$), $E^m = \int_{-\infty}^{m} tf(t)dt$ and $f(t)$ is the density function of LOS, with $\int_{-\infty}^{\infty} f(t)dt = 1$ and $f(t) \geq 0$. With $F(t) = \int_{-\infty}^{t} f(t)dt$ as the cumulative density function, $F(m)$ is the share of cases and $E^m / F(m)$ is the average LOS within the LOS threshold. The reimbursement scheme (2) includes an insurance contract covering LOS beyond the outlier threshold *m*. Assuming that the insurer loads the insurance premium by the factor *l* ($0 < l < 1$), with zero-profits, the insurer's per patient premium amounts to

$$p = E^m + (1+l)E_m , \tag{3}$$

where $E_m = \int_{m}^{\infty} tf(t)dt$ and $E_m / (1 - F(m))$ is the average LOS of the insured part of the LOS distribution.

The hospital's utility function is assumed to be

$$EU = \mu_\pi - \frac{r}{2}\sigma_\pi^2 , \tag{4}$$

with $r$ as the degree of absolute risk aversion ($r > 0$), $\mu_\pi$ as the expected profits per patient and $\sigma_\pi^2$ as the variance of profits per patient. If profits are normally distributed, (4) is a general representation of the preference of risk-averse agents (see *Meyer*, 1987).

The expected profits equal the difference between the expected reimbursements and the premium payments (3):

$$\begin{aligned} \mu_\pi &= E^m + E_m - p \\ &= -lE_m . \end{aligned} \tag{5}$$

The variance of profits is zero in the insured part of the LOS distribution. Given (2), the variance of the expected profits per patient, thus, amounts to:

$$\sigma_\pi^2 = \int_{-\infty}^{m} (E^m - t)^2 f(t)dt = \int_{-\infty}^{m} (t - E^m)^2 f(t)dt . \tag{6}$$

Inserting (5) and (6) into the expected utility function (4), yields:

$$EU(m) = -lE_m - \frac{r}{2}\int_{-\infty}^{m} (t - E^m)^2 f(t)dt . \tag{7}$$

Using the Leibniz rule and noting that $\partial E_m / \partial m = -\partial E^m / \partial m = -mf(m)$ holds, I derive for a marginal increase of the threshold:

$$\begin{aligned}
\frac{\partial EU}{\partial m} &= lmf(m) - \frac{r}{2}\left[\left(m - E^m\right)^2 f(m) + \int_{-\infty}^{m} 2\left(t - E^m\right)\left(-mf(m)\right) f(t)\,dt\right] \\
&= lmf(m) - \frac{r}{2} f(m)\left[\left(m - E^m\right)^2 - 2m\left(\int_{-\infty}^{m} tf(t)\,dt - E^m \int_{-\infty}^{m} f(t)\,dt\right)\right] \qquad (8) \\
&= mf(m)\left\{ l - \frac{r}{2}\left[\frac{\left(m - E^m\right)^2}{m} - 2E^m\left(1 - F(m)\right)\right]\right\},
\end{aligned}$$

An increase of the threshold, on the one hand, reduces the premium by $f(m)lm$, which in turn increases expected utility. On the other hand, it increases the profit risk by $f(m)\left[\left(m - E^m\right)^2 - 2mE^m\left(1 - F(m)\right)\right]$, which lowers expected utility.

Let

$$k(m) \doteq \frac{(m - E)^2}{m^*} - 2E^m\left(1 - F(m)\right) \qquad (9)$$

measure the change of the profit risk when the threshold marginally increases and rewrite (8) as:

$$\partial EU / \partial m = f(m)\, m\left(l - \frac{r}{2} k(m)\right). \qquad (10)$$

This equation illustrates the two key factors governing the tradeoff for an increased threshold: the loading factor *l*, determining the benefits, and the additional profit risk $k(m)$, capturing the costs. The optimal threshold $m^*$ balances the two opposite effects, giving rise to:

$$k\left(m^*\right) = \frac{2l}{r}. \qquad (11)$$

Assuming that a solution exists, (11) requires that $k\left(m^*\right) > 0$ since $l, r > 0$. The second-order condition for maximal expected utility is $\partial^2 EU / \partial m^2 < 0$. For $m = m^*$, $l = (r/2)k\left(m^*\right)$ and thus $\partial^2 EU / \partial m^2 = f\left(m^*\right) m^* \left(l - (r/2)\partial k\left(m^*\right)/\partial m\right)$. It follows that $\partial k\left(m^*\right)/\partial m > 0$ in the expected utility maximum, given $f(m), m, l, r > 0$.

*Proposition 1: The optimal threshold i) decreases with an increase of the degree of risk aversion r and ii) increases with an increase of the loading factor l.*

Proof: i) For infinitesimal small changes of *m* and *r* it holds around the maximum:

$$\left[\partial k\left(m^*\right)/\partial m\right]dm = -\left[\partial\left(l/r\right)/\partial r\right]dr \text{, or}$$

$$\frac{dm}{dr} = -\frac{l/r^2}{\partial k\left(m^*\right)/\partial m} < 0 \text{, since } \partial k\left(m^*\right)/\partial m > 0 \ . \ \blacksquare$$

ii) As $\left[\dfrac{\partial\left(l/r\right)}{\partial l}\right]dl = \dfrac{1}{r}dl > 0$, it follows that $\dfrac{dm}{dl} = \dfrac{1/r}{\partial k\left(m^*\right)/\partial m} > 0$. ■

With higher loading, the optimal LOS threshold increases. In other words, the hospital opts for a lower insurance coverage when the premium becomes more expensive. Furthermore, the threshold decreases when the degree of risk aversion increases. A risk-neutral hospital ($r = 0$), by comparison, would not choose any insurance at all (i.e. $m^* = \infty$).

I have assumed that the cost per diem of a stay is one. If the costs per diem are *β*, the effect of an increase of the threshold on expected utility becomes $\partial EU/\partial m = f\left(m\right)\beta m\left(l-\left(r/2\right)\beta k\left(m\right)\right)$ (see (9) and (10)). The optimal threshold, thus, requires:

$$k\left(m^*\right) = \frac{2l}{\beta r} \ . \tag{12}$$

*Proposition 2: The optimal threshold decreases with an increase of the cost per diem β.*

If the LOS distribution is the same across all DRGs, proposition 2 implies that $\beta m^*$ is the same for all DRGs. The current rule sets the upper threshold depending on the mean and the standard deviation of LOS (see (1)), but not on the costs of treatment. According to proposition 2, the optimal LOS threshold should c. p. decrease with the cost of treatment, reflecting the corresponding increase of the hospital's profit risk.

## 3. LOS standard deviation and the optimal threshold

In order to approach the German outlier rule, which sets the LOS threshold two standard deviations above the mean, I parameterize the distribution and assume a normally distributed LOS according to:

$$t = \mu + \sigma\varepsilon \text{, with } E(\varepsilon) = 0,\ Var(\varepsilon) = 1. \tag{13}$$

The density function of the normal distribution is

$$\begin{aligned} f(t) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-((t-\mu)/\sigma)^2/2} \\ &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\varepsilon^2/2} \quad . \end{aligned} \tag{14}$$

Since LOS is non-negative, I consider the normal distribution truncated below at point 0. The density function of the truncated distribution writes:

$$f_0(t) = \begin{cases} 0, & -\infty \le t \le 0 \\ \dfrac{f(t)}{1-F(0)}, & 0 \le t \le \infty \ , \end{cases} \tag{15}$$

where $F(0) = \int_{-\infty}^{0} f(t)dt$ and $f(t)$ is defined as in (14).

Using (13)-(15), I find:

$$\begin{aligned} E_0^m &= \int_0^m t f_0(t) dt = \frac{1}{\sqrt{2\pi}\left(1-F(0)\right)} \int_{-a/\sigma}^{(m-a)/\sigma} (\mu + \sigma\varepsilon) e^{-\varepsilon^2/2} d\varepsilon \\ &= \frac{1}{\sqrt{2\pi}\left(1-F(0)\right)} \left\{ \mu \int_{-a/\sigma}^{(m-a)/\sigma} e^{-\varepsilon^2/2} d\varepsilon - \sigma e^{-\varepsilon^2/2} \Big|_{-a/\sigma}^{(m-\mu)/\sigma} \right\} \\ &= \frac{\mu\left[F(m) - F(0)\right] - \sigma^2\left[f(m) - f(0)\right]}{1-F(0)} \quad . \end{aligned} \tag{16}$$

For the non-truncated distribution, the truncation is at $-\infty$. Since $F(-\infty) = f(-\infty) = 0$, it follows that $E^m = E_{-\infty}^m = \mu F(m) - \sigma^2 f(m)$.

With the truncated distribution, the optimal LOS threshold (see (9) and (11)) slightly changes to:

$$k\left(m^*\right) \doteq \frac{\left(m^* - E_0^{m^*}\right)^2}{m^*} - 2E_0^{m^*}\left(1 - F\left(m^*\right) + F(0)\right) = \frac{2l}{r}. \tag{17}$$

*Proposition 3: The optimal threshold decreases with an increase of the LOS standard deviation, provided the latter increases the marginal profit risk.*

Proof: At the optimal threshold, $\frac{dm}{d\sigma} = -\frac{\partial k\left(m^*\right)/\partial\sigma}{\partial k\left(m^*\right)/\partial m}$ holds. Since $\partial k\left(m^*\right)/\partial m > 0$, it follows that $sign\left[\frac{dm}{d\sigma}\right] = -sign\left[\frac{\partial k}{\partial\sigma}\right]$. ■

In order to calculate $\partial k\left(m^*\right)/\partial\sigma$, I need to derive $\partial E_0^{m^*}\left(m^*\right)/\partial\sigma$, as well as $\partial F/\partial\sigma$. From (16) I find:

$$\frac{\partial E_0^m}{\partial\sigma} = \frac{\mu\left[\frac{\partial F(m)}{\partial\sigma} - \frac{\partial F(0)}{\partial\sigma}\right] - 2\sigma\left[f(m) - f(0)\right] - \sigma^2\left[\frac{\partial f(m)}{\partial\sigma} - \frac{\partial f(0)}{\partial\sigma}\right] + \frac{\partial F(0)}{\partial\sigma}E_0^m}{1 - F(0)}. \tag{18}$$

Moreover, it holds that:

$$\frac{\partial F(t)}{\partial\sigma} = -f(t)\frac{t-\mu}{\sigma} \quad \text{and} \tag{19}$$

$$\frac{\partial f(t)}{\partial\sigma} = f(t)\frac{\left((t-\mu)/\sigma\right)^2 - 1}{\sigma}. \tag{20}$$

Inserting these equations into (18) leads to:

$$\frac{\partial E_0^m}{\partial\sigma} = \frac{-f(m)\left[\sigma^2 + m(m-\mu)\right] + f(0)\left(\sigma^2 + \mu E_0^m\right)}{\sigma\left[1 - F(0)\right]}. \tag{21}$$

Furthermore, from (17) I derive the effect of a change in the standard deviation on the hospital's additional risk at the LOS threshold:

$$\frac{\partial k}{\partial\sigma} = 2\left[\left(\frac{\partial F(m)}{\partial\sigma} - \frac{\partial F(0)}{\partial\sigma}\right)E_0^m - \left(2 - E_0^m/m - F(m) + F(0)\right)\frac{\partial E_0^m}{\partial\sigma}\right]. \tag{22}$$

This equation reveals two effects of an increase of $\sigma$ on the hospital's additional risk. First, a higher spread of the distribution changes the mass of LOS in the interval where the hospital bears the risk. (19) yields:

$$\frac{\partial F(m)}{\partial\sigma}-\frac{\partial F(0)}{\partial\sigma}=-\frac{m-\mu}{\sigma}f(m)-\frac{\mu}{\sigma}f(0). \tag{23}$$

Given $m>\mu$, the first effect of the derivative (22) is negative. This is because an increase of the standard deviation tends to lower the risk, as a higher share of LOS will be covered by insurance.

The second effect, however, likely cuts in the opposite direction. First, I observe $2-E_0^m/m-F(m)+F(0)>0$ as $0\le E_0^m/m, F(m), F(0)\le 1$, given $m>\mu$. Hence, the sign of the second effect has the inverse sign of $\partial E_0^m/\partial\sigma$. For the non-truncated distribution, I find $\partial E_0^m/\partial\sigma<0$ from (21). Hence, the second effect is indeed positive in this case. The hospital's marginal risk increases as the threshold moves further away from the mean when the standard deviation increases. For the truncated distribution, the second effect cannot be signed.

The total effect of an increase of the LOS standard deviation on the hospital's marginal risk is unclear since the difference of the two mentioned effects cannot be signed even when the distribution is non-truncated. Hence, I have to depend on simulations based on the factual distribution of LOS to evaluate the total effect.

*Figure 1* shows the value of the derivative (22) for truncated and non-truncated LOS as a function of the threshold. $\mu$ and $\sigma$ are three examples taken from the German top 30 DRG list (see Table 1).[2] For the non-truncated distributions, the derivative is positive, declining to zero for large thresholds. For both the truncated and the non-truncated distributions, the derivative is positive over the whole range of thresholds. When the threshold is close to the mean, an increase of the threshold raises the marginal profit risk. The maximum is attained within one to three days, depending on the LOS distribution characteristics. For larger thresholds the effect, driven by the density function, fast converges to zero.

Alternatively, one can evaluate the DRG threshold rule at the given thresholds and study whether the marginal risk increases with an increase of the standard deviation. The result of this is presented in two final columns of Table 1, showing the values of $\partial k/\partial\sigma$ for the current and the calculated German thresholds. Notice that except for two DRGs the sign of the derivative is positive. Thus, in almost all cases the hospital marginal risk increases, so that a risk-averse hospital would like to extend insurance coverage, i.e. to lower the LOS threshold.

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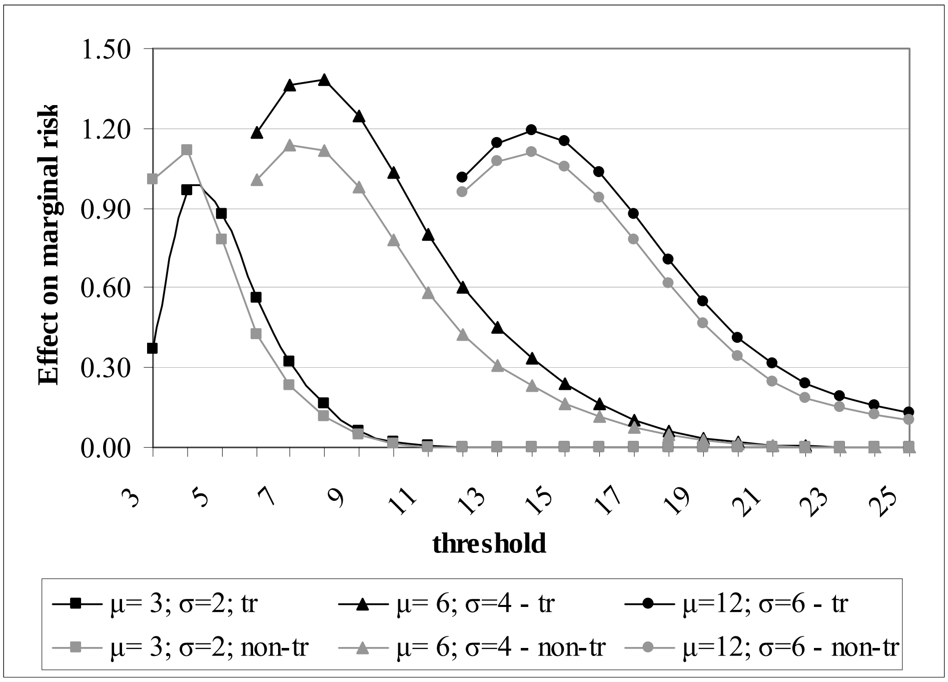
[2] If I interpret the observed mean as the mean of the truncated distribution ($E_0^\infty$), the mean of the non-truncated ($\mu$) can be derived using (16): $\mu=E_0^\infty-\sigma^2 f(0)/(1-F(0))$. If I employ $\mu$ to calculate the derivative (22), the results regarding the sign of (22) remain unchanged.

**Figure 1**: **The effect of marginal profit risk from an increase of *σ* for different *μ* and *σ*; truncated (tr) and non-truncated (non-tr) normal distribution**




## 5. Discussion

Since a hospital can influence a patient's LOS, the assumption of an exogenous LOS is unrealistic. An endogenous LOS raises problems of moral hazard. Beyond the threshold, reducing the LOS lowers premium loading payments by $dE_m l$, while within the threshold a reduction of the LOS, $dE^m$, translates one to one in higher profits. In other words, insurance coverage dilutes the incentive to reduce the LOS. The optimal policy then needs to approach the trade-off between risk spreading and appropriate incentives (*Zeckauser*, 1979), which can be solved by combining a threshold with a coinsurance on the marginal costs of stay beyond the threshold. DRG systems reflect this, as outlier days are usually reimbursed with a 40% rebate on the average cost per diem.

An endogenization of the LOS will not necessarily change the comparative statics for the optimal threshold regarding the variance of the LOS. Consider first the case where the productivity of efforts to reduce the LOS is independent of the initial LOS. Abstracting from the discontinuity at the threshold, efforts will then only shift the LOS distribution to the left and have no effect on the LOS variance. Consequently, although the optimal threshold will increase, the qualitative relationship between the standard deviation and the optimal threshold will not change. A higher LOS variance c. p. will come with a decrease of the optimal threshold.

The productivity of efforts to reduce LOS may, however, increase with the initial LOS, so that efforts will have an effect on the LOS variance. Still, it appears that the following hierarchy of instruments would apply in this case. Moral hazard can be restrained using a non-linear coinsurance rate that increases with the observed LOS to give a stronger incentive to reduce the LOS when it is less expensive to do so. A higher LOS variance will have no effect on the efforts to reduce LOS. The optimal threshold will reflect the efforts to reduce LOS but the comparative statics regarding the variance of LOS will not change its qualitative nature. To prove this conjecture would be difficult, given the discontinuity of the LOS distribution, arising when efforts to reduce LOS are introduced.

*Ellis and McGuire* (1988) criticized the existing outlier payments based on individual cases and proposed an insurance scheme based on the average case. Risk pooling within the hospital will reduce the variance of the profit per patient and, thus, decrease insurance demand. This qualification, however, does not affect the optimal threshold rule. A hospital which shoulders a higher risk due to a large case-mix index c. p. will demand a lower threshold compared to a hospital with a lower case-mix load.

The new generation of outlier payment systems in the USA is no longer based on the LOS, but on the patients' costs of stay[3]. This reflects the empirical observation that after controlling for DRG, the costs of stay are only weakly related to LOS among very long stay cases (see *Keeler et al.*, 1988). Interestingly, the cost outlier schemes do not define the thresholds as a function of the variance. Rather, a cost-to-charge factor determines the threshold. In this case a mean-preserving increase in the standard deviation will not affect the threshold. This rule is better than the former one that sets the threshold two standard deviations above the mean.

Australian outlier payments do not depend on parametric distribution, arguing that the LOS is not normally distributed.[4] The threshold, called the high trim point, is often 2 or 3 times the average length of stay. Like the US Medicare cost outlier, this scheme appears to dominate the original threshold rule, as it does not further aggravate the hospitals' profit risk, by increasing the threshold when the LOS standard deviation increases.

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[3] See for instance, Department of Health and Human Services, Centers for Medicare and Medicaid Services, 42 CFR Part 412, Federal Register, Vol. 68, No. 43, March 5, 2003, p. 10420-10429.

[4] For New South Wales, see http://www.health.nsw.gov.au/, for Victoria, see http://www.health.vic.gov.au/pfg/, and for South Australia, see http://www.health.sa.gov.au/.

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