Insights into Clients' Choice in Preventive Health Care Facility Location Planning

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Clients' Choice Behavior

- N set of clients
- M choice set: set of alternatives clients choose from
- L set of attributes (related to $m \in M$) or characteristics (related to $n \in N$)

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Assumptions (Train, 2003)

- M is exhaustive and alternatives are mutually exclusive
- $n \in N$ chooses exactly one alternative from choice set M
- $n \in N$ chooses alternative $j \in M$ that maximizes utility u_{nj} of alternative j for client n

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$\Rightarrow$ **utility maximization choice rule**, i.e. n chooses j, iff

$$u_{nj} > u_{nm} \; \forall \; m \in M, m \neq j. \tag{1}$$

Operationalization of Utility

We cannot observe all attributes & characteristics: **stochastic utility**

$$u_{nj} = v_{nj} + \epsilon_{nj} \tag{2}$$

with

v_{nj} deterministic part of utility

ϵ_{nj} stochastic part of utility

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Assume v_{nj} to be linear in parameters:

$$v_{nj} = \sum_{l \in L} \beta_{jl} c_{njl} \tag{3}$$

c_{njl} value of attribute/characteristic l concerning individual n and alternative j

β_{jl} utility contribution per unit of attribute l related to alternative j ($\leftarrow$MLE)

Client Choice Model

Stochastic utility:

$$p_{nj} = \text{Prob}\left(u_{nj} > u_{nm} \ \forall \ m \in M, m \neq j\right) \tag{4}$$

Assume ϵ_{nj} iid EV:

Multinomial Logit Model (MNL)

$$p_{nj} = \frac{e^{v_{nj}}}{\sum_{m \in M} e^{v_{nm}}} \tag{5}$$

denotes probability that client n chooses alternative j.

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Choice probability of clients located in $i \in I$ choosing $j \in M$ is given by the MNL

$$p_{ij} = \frac{e^{v_{ij}}}{\sum_{m \in M} e^{v_{im}}} \tag{7}$$

Note, $\sum_{j \in J} p_{ij} < 1$, while $\sum_{j \in M} p_{ij} = 1$

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$\overline{p}_{ij}$ maximum choice probability of clients located in $i \in I$ patronizing a facility located in $j \in J$: $\overline{p}_{ij} = e^{v_{ij}} / (e^{v_{ij}} + e^{v_{i0}})$; 0 denotes the " no choice"

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F^A objective function value: expected participation in preventive health care

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F^A objective function value: expected participation in preventive health care

x_{ij} choice probability of clients at node $i \in I$ access health care service at location $j \in J$

$$x_{ij} = \frac{e^{v_{ij}} y_j}{e^{v_{i0}} + \sum_{m \in J} e^{v_{im}} y_m} \quad \forall\ i \in I, j \in J \tag{8}$$

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- p_{ij}/p_{ik} is independent from a third alternative $m \in M$, hence if facilities are located at j and k

$$p_{ij}/p_{ik} = x_{ij}/x_{ik} \tag{9}$$

Model

$$\text{Maximize } F^A = \sum_{i \in I} g_i \sum_{j \in J} x_{ij} \tag{10}$$

subject to

$$\widetilde{x}_i + \sum_{j \in J} x_{ij} \leq 1 \qquad \forall\, i \in I \tag{11}$$

$$x_{ij} \leq \overline{p}_{ij}\, y_j \qquad \forall\, i \in I, j \in J \tag{12}$$

$$\widetilde{p}_i x_{ij} \leq p_{ij} \widetilde{x}_i \qquad \forall\, i \in I, j \in J \tag{13}$$

$$\widetilde{p}_i x_{ij} \geq p_{ij} \widetilde{x}_i + y_j - 1 \qquad \forall\, i \in I, j \in J \tag{14}$$

$$\sum_{i \in I} g_i x_{ij} \geq R_{\min}\, y_j \qquad \forall\, j \in J \tag{15}$$

Model continued

$$\sum_{i \in I} g_i x_{ij} \leq \sum_{k=1}^{K} q_k w_{jk} \qquad \forall\, j \in J \tag{16}$$

$$\sum_{k=1}^{K} w_{jk} = y_j \qquad \forall\, j \in J \tag{17}$$

$$\sum_{j \in J} \sum_{k=1}^{K} k \cdot w_{jk} \leq Q_{\max} \tag{18}$$

$$x_{ij} \geq 0 \qquad \forall\, i \in I, j \in J \tag{19}$$

$$\tilde{x}_i \geq 0 \qquad \forall\, i \in I \tag{20}$$

$$y_j \in \{0, 1\} \qquad \forall\, j \in J \tag{21}$$

$$w_{jk} \in \{0, 1\} \qquad \forall\, j \in J, k = 1, \ldots, K \tag{22}$$

Linear Reformulation by Zhang et al (2012)

- $v_{i,\text{no choice}} = 0$

M_1 big number; $= 1$ for example

M_2 big number; $= 1$ for example

z_{ijo} artificial continuous variable for avoiding non-linearity with $o \in J$

$$x_{ij} + \sum_{o \in J} e^{v_{io}} z_{ijo} = e^{v_{ij}} y_j \qquad i \in I, j \in J \qquad (23)$$

$$z_{ijo} \leq x_{ij} \qquad i \in I, j, o \in J \qquad (24)$$

$$z_{ijo} \leq M_1 y_j \qquad i \in I, j, o \in J \qquad (25)$$

$$z_{ijo} \geq x_{ij} - M_2(1 - y_j) \qquad i \in I, j, o \in J \qquad (26)$$

$$z_{ijo} \geq 0 \qquad i \in I, j, o \in J \qquad (27)$$

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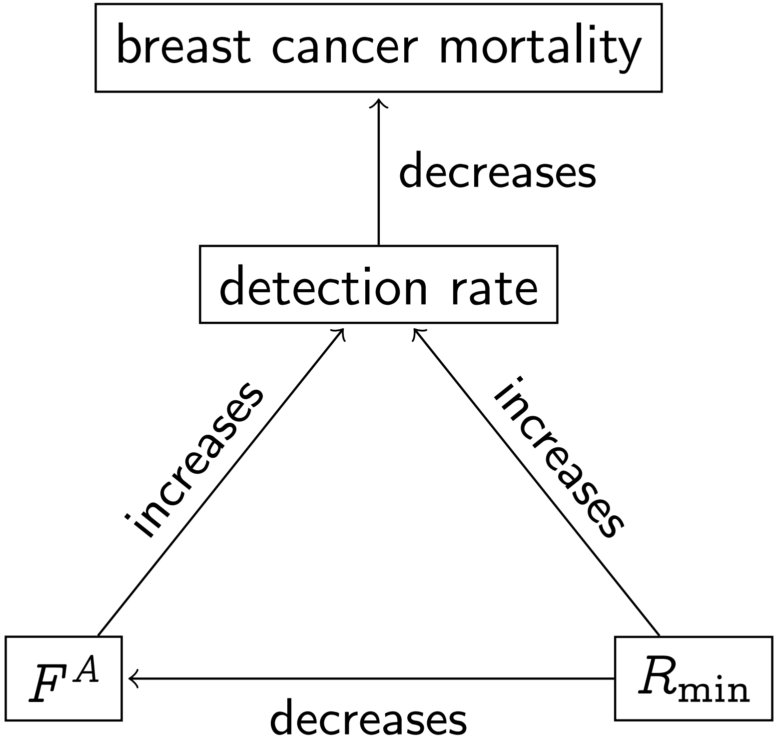
  - participation (F^A)
  - quality of examination ($R_{\min}$)
- Negative trade-off between F^A and exogenously given $R_{\min}$

Trade Off



- Improvement: make $R_{\min}$ endogenous to clients' choice behavior

Numerical Example

- $j \in \{A, B, C\}$
- $Q_{\text{max}} = 2$
- R_{min} be given such that $x_{1j} > 0.15$ is required

α_j true-positive-rate

	v_{1j}	$\{B, C\}$		$\{A, C\}$	
		x_{1j}	α_j	x_{1j}	α_j
facility $j = A$	-1.5	0.000		0.154	0.9
facility $j = B$	-1.0	0.231	0.9	0.000	
facility $j = C$	-1.5	0.140	0.8	0.154	0.9
no-choice $\tilde{x}_1$, $j = 0$	0	0.629		0.691	
participation rate	-	0.371		0.309	
detection rate	-	0.320		0.276	

poor $\alpha_C = 0.5$, solution $\{B, C\}$ yields a better detection rate than solution $\{A, C\}$

Contents

Artificial Data Generation

1. Nodes:
   - Set $\mid I \mid$ and $\mid J \mid$
2. Population given $i \in I$
   2.1 Select randomly $\phi_i \in [0, 2.4]$
   2.2 $g_i := \phi_i \cdot |J| \,/\, |I|$
3. Travel times, utility, choice probabilities given $i \in I$ and $j \in J$
   3.1 Generate randomly Cartesian coordinates of nodes in the interval $[0, 100]$ (uniform distribution)
   3.2 t_{ij} := rectangle distance from node $i \in I$ to facility $j \in J$ divided by 60
   3.3 $v_{ij} := -\beta \cdot t_{ij}$ with $\beta = 2$
   3.4 $v_{i0} := 0$
   3.5 $p_{ij} := \frac{e^{v_{ij}}}{\sum_m e^{v_{im}}}$
   3.6 $\widetilde{p}_i := \frac{1}{1+\sum_j e^{v_{ij}}}$
   3.7 $\bar{p}_{ij} := \min\{0.5, e^{v_{ij}}/(1 + e^{v_{ij}})\}$

Artificial Data Generation (continued)

Other parameters according to Zhang et al. (2012)

- $K := 4$
- $\nabla\lambda_k := 1.05^k$
- $R_{\min} := 1.2$
- $q_k := B \cdot \nabla\lambda_k$ with $B = 1.5 \cdot R_{\min}$
- $Q_{\max} := \lceil(|J|/2)\rceil$
- $M_1 = M_2 := 1$

Results

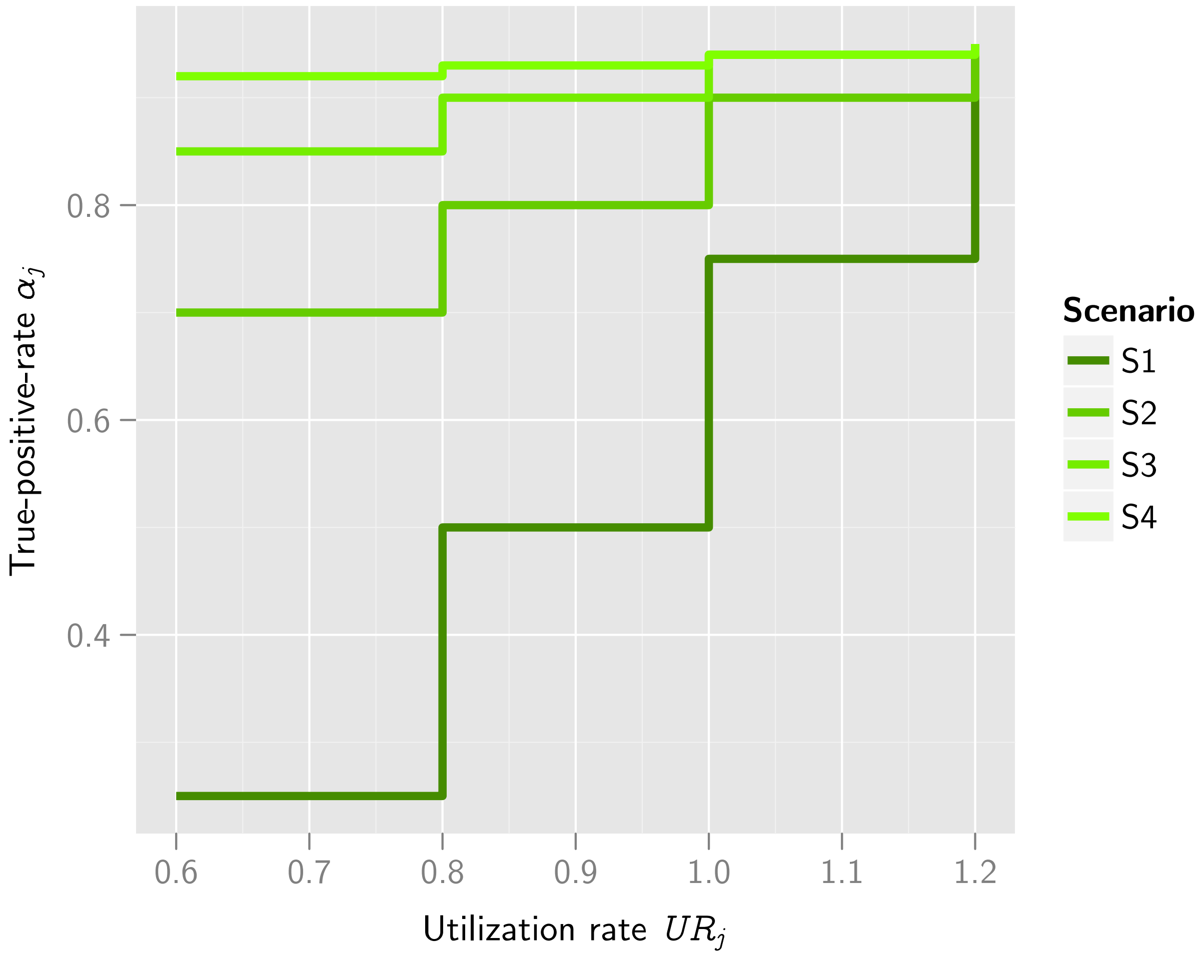
$\lvert J\rvert$	$\lvert I\rvert$	Model Z		Model A		Model A with $\underline{F}^A = LB^*$		
		"GAP"	CPU	"GAP"	CPU	"GAP"	CPU (LB^*)	$100\frac{\underline{F}^{A*}-LB^*}{\underline{F}^{A*}}$
10	100	0	35	0	4	0	9 (2)	1.55
	200	0	196	0	15	0	23 (2)	4.18
	400	0	831	0	60	0	51 (7)	2.97
20	100	406.44[b]	3600	0	619	0	137 (1)	2.48
	200	-	3600	1.66	2858	0	604 (2)	1.66
	400	185.34[d]	3600	7.35	3600	0.22	1993 (2)	0.93[a]
40	100	89.99[c]	3600	8.58	3600	3.34	3600 (8)	-
	200	96.44[c]	3600	27.88	3600	8.21	3600 (25)	-
	400	92.69[d]	3600	55.67	3600	12.10	3600 (75)	-

Number of instances where an optimal solution is obtained: a: 9, b: 5, c: 3, d: 1

- Averages over ten random instances
- CPU: CPLEX time in seconds
- (): time to compute LB
- GAP provided by CPLEX

Numerical Investigation of Trade-Off between F^A and $R_{\min}$

Utilization rate $UR_j = \sum_{i \in I} g_i x_{ij} \; \forall \, j \in J$,



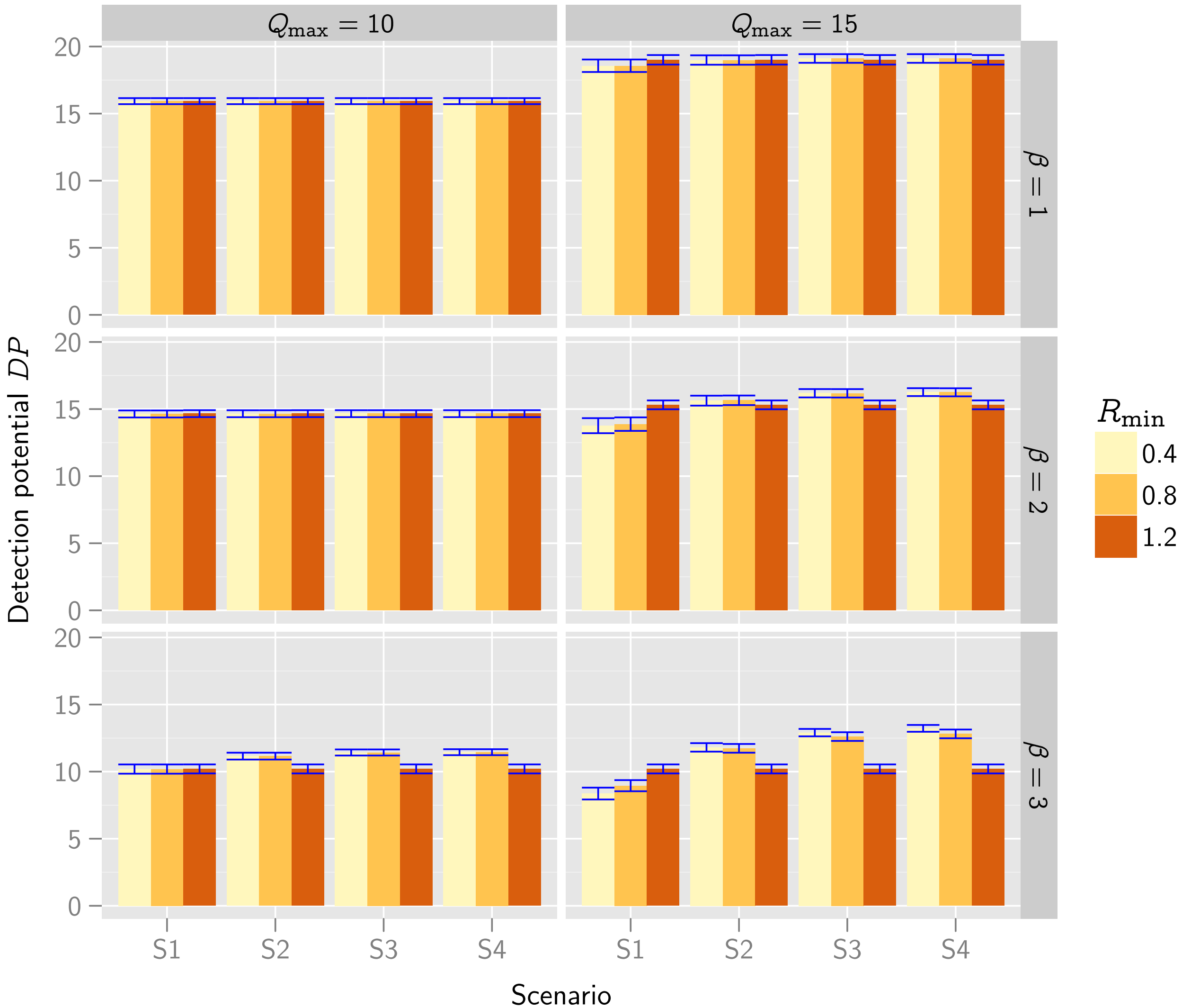
Data

Total detection potential

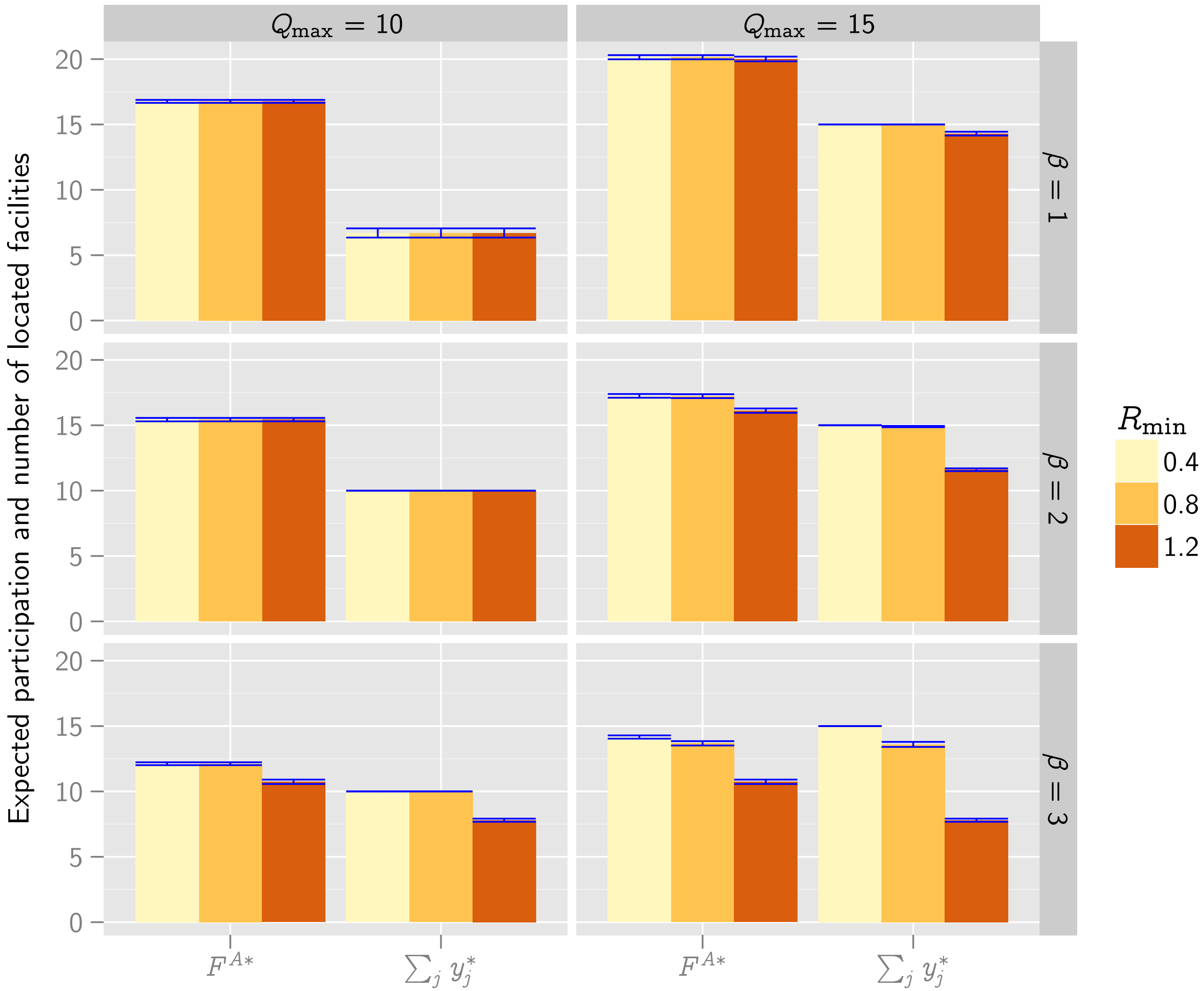
$$DP = \sum_{i \in I} g_i \sum_{j \in J} \alpha_j x_{ij} \tag{28}$$

- $|J| = 20$, $|I| = 100$
- $B = 1.2 \cdot 1.5 = 1.8$
- 10 random instances for $R_{\min} = \{1.2, 0.8, 0.4\}$

Resulting Detection Potential



Expected Participation



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$\Rightarrow$ incorporation of expected waiting times and quality of care (true-positive rate, for example) in clients' utility function

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Contents

Step 1

Ω_{ij} = 1, if node $i \in I$ is assigned to facility location $j \in J$ (0, otherwise),

Ψ_j = 1, if location $j \in J$ provides a health care facility (0, otherwise),

F^B ofv indicating the cumulative choice probabilities,

F^C ofv indicating the attractiveness of the located facilities, and

LB lower bound to Model A.

Model B

$$\text{Maximize } F^B = \sum_{i \in I} \sum_{j \in J} p_{ij} \Omega_{ij} \tag{29}$$

subject to

$$\sum_{j \in J} \Omega_{ij} = Q_{\max} \quad \forall\, i \in I \tag{30}$$

$$\Omega_{ij} \in \{0, 1\} \quad \forall\, i \in I, j \in J \tag{31}$$

determines for each demand node i the $Q_{\max}$ most attractive locations.

Step 2

Attractiveness

$$b_j = \sum_{i \in I} g_i \Omega^*_{ij} \; \forall \, j \in J \tag{32}$$

Model C

$$\text{Maximize } F^C = \sum_{j \in J} b_j \Psi_j \tag{33}$$

subject to

$$\sum_{j \in J} \Psi_j = Q_{\max} \tag{34}$$

$$\Psi_j \in \{0, 1\} \qquad \forall \, j \in J \tag{35}$$

determines the $Q_{\max}$ most attractive facility locations.

Step 3

Model D

$$\text{Maximize } LB = \sum_{i \in I} g_i \sum_{j \in J} x_{ij} \quad (36)$$

subject to (11) - (22) and

$$y_j \leq \Psi_j^* \quad \forall\, j \in J \quad (37)$$

determines a lower bound to Model A